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# Forecasting with Global Vector Autoregressive Models: A Bayesian Approach\*

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## Abstract

This paper develops a Bayesian variant of global vector autoregressive (B-GVAR) models to forecast an international set of macroeconomic and financial variables. We propose a set of hierarchical priors and compare the predictive performance of B-GVAR models in terms of point and density forecasts for one-quarter-ahead and four-quarters-ahead forecast horizons. We find that forecasts can be improved by employing a global framework and hierarchical priors which induce country-specific degrees of shrinkage on the coefficients of the GVAR model. Forecasts from various B-GVAR specifications tend to outperform forecasts from a naive univariate model, a global model without shrinkage on the parameters and country-specific vector autoregressions.

JEL Classification: *C11, C32, C53, C55, F44.*

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## 1 Introduction

The rise in international trade and cross-border financial flows in recent decades implies that countries are more than ever exposed to economic shocks from abroad, as demonstrated by the recent global financial crisis. Hence, macroeconomic tools that treat countries as isolated from the rest of the world may miss important information for forecasting and counterfactual analysis. Such concerns do not arise with global vector autoregressive (GVAR) models, as they accommodate spillovers from the global economy in a systematic and transparent manner. The GVAR framework consists of single-country models that are stacked to yield a comprehensive representation of the world economy.

The empirical literature on GVAR models has been largely influenced by the work of M. Hashem Pesaran and co-authors (Pesaran *et al.*, 2004; Garrat *et al.*, 2006; Dees *et al.*, 2007b;a). In a series of papers, these authors examine the effect of US macroeconomic shocks on selected foreign economies employing either generalized impulse response functions, structural identification schemes or overidentifying restrictions on long-run relationships between macroeconomic variables to identify the shocks (Pesaran *et al.*, 2004; Dees *et al.*, 2007b;a). Recent papers have advanced the literature on GVAR modeling in terms of country coverage (Feldkircher, 2015), identification of shocks (Eickmeier & Ng, 2015) and the specification of international linkages (Chudik & Fratzscher, 2011; Eickmeier & Ng, 2015; Feldkircher & Huber, 2015; Galesi & Sgherri, 2013).

Most of the existing applications of GVAR models concentrate on the quantitative assessment of the propagation of macroeconomic shocks using historical data, while very few contributions have addressed their forecasting performance. Evaluating GVAR forecasts in an out-of-sample exercise, Pesaran *et al.* (2009) propose to pool GVAR forecasts over different estimation windows and model specifications in order to account for potential structural breaks and misspecifications. Pesaran *et al.* (2009) conclude that taking global links across economies into account using GVAR models leads to more accurate out-of-sample predictions than using forecasts based on univariate specifications for output and inflation. Yet for interest rates, the exchange rate and financial variables, the results are less spectacular, and the authors also find strong cross-country heterogeneity in the performance of GVAR forecasts. Employing a GVAR model to forecast macroeconomic variables in five Asian economies, Han & Ng (2011) find that one-step-ahead forecasts from GVAR models outperform those of stand-alone VAR specifications for short-term interest rates and real equity prices. Concentrating on predicted directional changes to evaluate the forecasting performance of GVAR specifications, Greenwood-Nimmo *et al.* (2012) confirm the superiority of GVAR specifications over univariate benchmark models at long-run forecast horizons.

As an alternative to the GVAR framework a related strand of the literature advocates the estimation of large VARs or panel VARs using Bayesian techniques. More specifically,

Bańbura *et al.* (2010) assess the forecasting performance of a large-scale monetary VAR based on more than 100 macroeconomic variables and sectoral information. They show that forecasts of these large-scale models can outperform small benchmark VARs when the degree of shrinkage on the parameters is set accordingly. Giannone & Reichlin (2009) and Alessi & Bańbura (2009) propose to exploit these shrinkage properties and estimate Bayesian VARs with a large cross-section of countries. Alessi & Bańbura (2009) show that Bayesian VAR specifications as well as dynamic factor models are able to yield accurate one-quarter to four-quarters-ahead forecasts for international macroeconomic data. Koop & Korobilis (2014) propose a panel VAR framework that overcomes the problem of overparametrization by averaging over different restrictions on interdependencies between and heterogeneities across cross-sectional units. More recently, Korobilis (2015) advocate a particular class of priors that allows for soft clustering of variables or countries, arguing that classical shrinkage priors are inappropriate for panel VARs.

In this contribution, we propose using established shrinkage priors and develop a Bayesian GVAR (B-GVAR) model. Akin to the GVAR framework, we assume that links among economies are determined exogenously, while we borrow strength from the Bayesian literature in estimating the individual country models. This allows us to keep the virtues of the GVAR framework with regard to offering a coherent way for policy and counterfactual analysis. Our model includes standard variables that are often employed in small-country VARs such as output, inflation, short-term and long-term interest rates, the real exchange rate, equity prices and the oil price as a global control variable (see e.g., Dees *et al.*, 2007b;a; Pesaran *et al.*, 2004; 2009, among others). This set of variables is extended to feature total credit (domestic and cross-border credit), which can act as an important transmission channel of international shocks.

We compare forecasts of the B-GVAR model under prior specifications that resurface frequently in Bayesian VAR empirical studies: the conjugate Minnesota prior (Litterman, 1986) and its non-conjugate version, and a weighted average of a Minnesota type prior, the “initial dummy observation” prior, which accommodates potential cointegration relationships among the variables considered, and the “single unit root” prior, which facilitates soft-differencing (Doan *et al.*, 1984; Sims, 1992; Sims & Zha, 1998). We extend this set of random-walk priors to include the stochastic search variable selection (SSVS) prior proposed by George *et al.* (2008) for VAR models. Since the hyperparameters for all priors are elicited locally (i.e., for the country model), our approach induces country-specific degrees of shrinkage on the parameters, which is expected to improve forecasts significantly. By inheriting the properties of their single-country VAR counterparts, B-GVAR models are expected to be less prone to overfitting (Giannone & Reichlin, 2009) and allow the researcher to include prior beliefs in the model, while still taking the long-run co-movement of variables into account. We compare our battery of priors using an expanding window to forecast developments one-quarter-ahead and

four-quarters-ahead. These forecasts are benchmarked to forecasts of a fifth-order autoregressive model with drift term by means of root mean squared errors for point forecasts, and log predictive scores for density forecasts. As another competitor, and to assess the importance of international linkages for forecasting, we evaluate forecasts from isolated, country-specific Bayesian vector autoregressions.

Our analysis provides new insights to the specification and estimation of global macroeconomic models. First, we find that forecasts can be improved by employing a global framework that allows for country-specific degrees of shrinkage on the parameters. The proposed Bayesian specifications of the GVAR tend to improve upon forecasts from the naive model, a global model without shrinkage and a shrinkage model that neglects international linkages. Second, we find that the prior specification put forward in [Sims & Zha \(1998\)](#), the non-conjugate Minnesota prior and the SSVS prior all show a strong forecasting performance. Third, our analysis indicates that Latin American variables are particularly hard to forecast, while the forecast performance for developed economies is more homogeneous among the specifications considered.

The paper is structured as follows. [Section 2](#) provides a brief description of the global VAR model, while [Section 3](#) derives its Bayesian variant. In [Section 4](#) we present the data and perform the forecast evaluation exercise. Finally, [Section 5](#) concludes.

## 2 The GVAR Model

GVAR specifications constitute a compact representation of the world economy designed to model multilateral dependencies among economies across the globe. Basically, a GVAR model consists of a number of country-specific specifications that are combined to form a global model.

The first step is to estimate separate multivariate time series models. In our case, these are standard vector autoregressive models involving a set of endogenous variables and enlarged by weakly exogenous and global control variables (VARX\* model). Assuming that our global economy consists of  $N + 1$  countries, we estimate a VARX\* of the following form for every country  $i = 0, \dots, N$ ,

$$x_{it} = a_{i0} + \sum_{s=1}^p \Phi_{is} x_{it-s} + \sum_{r=0}^{p^*} \Lambda_{ir} x_{it-r}^* + \varepsilon_{it}, \quad (2.1)$$

where  $x_{it}$  is a  $k_i \times 1$  vector of endogenous variables in country  $i$  at time  $t \in 1, \dots, T$ ,  $a_{i0}$  is a  $k_i$ -dimensional vector of intercept terms,  $\Phi_{is}$  ( $s = 1, \dots, p$ ) denotes the  $k_i \times k_i$  matrix of parameters associated with the lagged endogenous variables and  $\Lambda_{ir}$  ( $r = 1, \dots, p^*$ ) are the coefficient matrices of the  $k_i^*$  weakly exogenous variables, which are of dimension  $k_i \times k_i^*$ .

Furthermore,  $\varepsilon_{it}$  is the standard zero-mean vector error term with variance-covariance matrix  $\Sigma_{\varepsilon i}$ .

The weakly exogenous or *foreign* variables,  $x_{it}^*$ , are constructed as a weighted average of their cross-country counterparts,

$$x_{it}^* = \sum_{j=0}^N \omega_{ij} x_{jt}, \quad (2.2)$$

with  $\omega_{ij}$  denoting the (non-negative) weight corresponding to the pair of country  $i$  and country  $j$ . We assume that  $\omega_{ii} = 0$  and  $\sum_{j=0}^N \omega_{ij} = 1$ . The weights  $\omega_{ij}$  reflect economic and financial ties among economies, which are usually approximated using data on (standardized) bilateral trade flows.<sup>1</sup> The assumption that the  $x_{it}^*$  variables are weakly exogenous at the individual level reflects the belief that most countries are small relative to the world economy.

Following Pesaran *et al.* (2004) we stack the  $N + 1$  country-specific models to obtain a global model, which is given by

$$Gx_t = a_0 + \sum_{q=1}^Q H_q x_{t-q} + \epsilon_t. \quad (2.3)$$

Here,  $G$  is a  $k \times k$ -dimensional matrix that establishes contemporaneous relations between countries, with  $k = \sum_{i=0}^N k_i$ . Furthermore, let  $a_0$  be a  $k$ -dimensional vector associated with the constant and  $H_q (q = 1, \dots, Q)$  is a  $k \times k$ -dimensional global coefficient matrix (with  $Q = \max(p, p^*)$ ). The matrices  $G, a_0$  and  $H_q$  are complex functions of the corresponding country-specific parameters and the bilateral weights. Finally,  $\epsilon_t$  is a global vector error term with variance-covariance matrix  $\Sigma_{\epsilon}$ . Further details on the derivation of the GVAR model can be found in Appendix B.

### 3 The B-GVAR: Priors over Parameters

Bayesian analysis of the GVAR model requires the elicitation of prior distributions for all parameters of the model. We use several prior structures that have been developed for VAR specifications over the parameters of the individual country-specific models, which we extend to account for the presence of (weakly) exogenous variables.<sup>2</sup> For prior implementation, it proves convenient to rewrite the model in (2.1) as

$$x_{it} = \Pi_i' Z_{it-1} + \varepsilon_{it}, \quad (3.1)$$

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<sup>1</sup>See e.g., Eickmeier & Ng (2015) and Feldkircher & Huber (2015) for an application using a broad set of different weights.

<sup>2</sup>Karlsson (2012) provides an excellent overview for Bayesian VAR models.

where  $Z_{it-1} = (1, x'_{it-1}, \dots, x'_{it-p}, x'^*_{it}, \dots, x'^*_{it-p^*})'$  is of dimension  $K_i \times 1$ , where  $K_i = 1 + k_i p + k_i^*(p^* + 1)$  and  $\Pi_i = (a_{i0}, \Phi_{i1}, \dots, \Phi_{ip}, \Lambda_{i0}, \dots, \Lambda_{ip^*})'$  denotes a  $K_i \times k_i$  matrix of stacked coefficients. Up to this point we have not adopted any distributional assumptions for  $\varepsilon_{it}$ . We complete the model specification by assuming that the errors  $\varepsilon_{it}$  are multivariate Gaussian, i.e.,  $\varepsilon_{it} \sim \mathcal{N}(0, \Sigma_{\varepsilon i})$ .

Rewriting the model in terms of full-data matrices yields

$$x_i = Z_i \Pi_i + \varepsilon_i \quad (3.2)$$

where  $x_i$  is a  $T \times k_i$  matrix of stacked endogenous variables,  $Z_i$  is a  $T \times K_i$  matrix of stacked explanatory variables and  $\varepsilon_i$  is a  $T \times k_i$  matrix of errors. Furthermore, let  $\Psi_i = \text{vec}(\Pi_i)$  denote the  $v_i$ -dimensional coefficient vector with  $v_i = k_i K_i$ .

### The General Conjugate Prior Setup

We start with the simplest prior for the coefficients of the country-specific VARX\* models, which is the natural conjugate prior. In the VARX\* framework, we impose an inverted Wishart prior on  $\Sigma_{\varepsilon i}$  and a multivariate Gaussian prior on  $\Psi_i$

$$\Psi_i | \Sigma_{\varepsilon i} \sim \mathcal{N}(\underline{\Psi}_i, \Sigma_{\varepsilon i} \otimes \underline{V}_i), \quad (3.3)$$

$$\Sigma_{\varepsilon i} \sim \mathcal{IW}(\underline{S}_i, \underline{v}_i), \quad (3.4)$$

where  $\underline{\Psi}_i$  and  $\underline{V}_i$  denote prior mean and variance, respectively. Additionally, we let  $\underline{S}_i$  denote the prior scale matrix and  $\underline{v}_i$  the prior degrees of freedom for  $\Sigma_{\varepsilon i}$ . The natural conjugate prior possesses two convenient properties. First, the prior dependence between  $\Psi_i$  and  $\Sigma_{\varepsilon i}$  allows us to exploit a Kronecker factorization of the likelihood, which translates into significant computational advantages. However, it is worth noting that the Kronecker factorization implies prior variances on the coefficients that are proportional across equations of the country model, which might be very restrictive. Second, analytical results for the marginal posterior distributions (and functions thereof) are readily available. Especially for forecasting applications where the model has to be re-estimated several times over a training sample, this proves to be a significant advantage.

Following the literature on Bayesian VARs (Litterman, 1986; Sims, 1992; Sims & Zha, 1998), the most common choices for  $\underline{\Psi}_i$  and  $\underline{V}_i$  are given by the so-called random walk priors. Under the prior, the variables in the system are assumed to follow simple random walks. To implement this prior, we set the prior mean according to

$$\underline{\Psi}_{ij} = \begin{cases} \underline{a}_{ij} & \text{for the first own lag of endogenous variable } j \text{ in equation } j \\ 0 & \text{in all other cases.} \end{cases} \quad (3.5)$$

where  $\underline{a}_{ij}$  ( $j = 1, \dots, k_i$ ) refers to the prior mean over the parameter associated with the first own lag of the  $k_i$  endogenous variables. Usually these are set to one for variables in levels, leading to the traditional random walk prior. The assumption that the endogenous variables a priori follow random walk processes at the local level directly carries over to the global model. To see this, note that under the prior model, the coefficients associated with the contemporaneous and lagged (weakly) exogenous variables are set equal to zero. Moreover, the coefficients corresponding to higher lag orders of the endogenous variables are also set equal to zero. Hence, the  $G$  and  $H$  matrices reduce to  $k \times k$  identity matrices. Consequently, the global prior model is given by

$$x_t = x_{t-1} + e_t, \quad (3.6)$$

where it is straightforward to show that the variance-covariance matrix of  $e_t$  is a block-diagonal matrix with the corresponding  $i$ th block being equal to the prior expectation of  $\Sigma_{\varepsilon i}$ . The only assumption which is crucial for this result to hold is that the prior mean of coefficients related to the weakly exogenous variables is set to zero.

Several choices are recommended in the literature for the elicitation of  $\underline{V}_i$ , which translate into different assumptions about the behavior of the prior model. [Doan \*et al.\* \(1984\)](#), [Kadiyala & Karlsson \(1997\)](#) and [Sims & Zha \(1998\)](#) propose three prominent prior specifications that have been frequently employed by practitioners. The most prominent prior is the Minnesota prior, which has a proven track record in terms of forecasting performance. The Minnesota prior specifies the prior variance on the coefficients,  $\underline{V}_i$ , such that

$$\underline{V}_{ig,l} = \begin{cases} \frac{\alpha_{i1}}{r^\kappa \sigma_{ig}} & \text{for variance on the } r\text{th lag of coefficient attached to variable } g \\ \frac{\alpha_{i2}}{(1+r)^\kappa \sigma_{ig}^*} & \text{for variance on the } r\text{th lag of coefficient attached to weakly exog. variable } g \\ \alpha_{i3} & \text{for variances on the deterministic part of the model.} \end{cases} \quad (3.7)$$

Here, hyperparameters  $\alpha_{i1}$  and  $\alpha_{i2}$  control the tightness of the prior on the endogenous and weakly exogenous part, respectively. Moreover, the priors are scaled using standard deviations obtained by running univariate autoregressions on the particular variables. Specifically,  $\sigma_{ig}$  refers to the standard deviation of a univariate autoregressive model for the corresponding variable, whereas  $\sigma_{ig}^*$  denotes the standard deviation obtained from an autoregressive model of the  $l$ th weakly exogenous variable. Finally,  $r^\kappa$  is a deterministic function of the lag length. Consequently, the strength of the prior belief in the random walk specification is governed by  $\alpha_1$ . The hyperparameter  $\kappa$  increasingly tightens the variance on the prior for distant lags, reflecting the belief that specifications including non-zero parameters associated to variables with long lags tend to have a detrimental effect on the forecasting performance of the model.



There is a direct link between the locally specified  $\underline{V}_i$  and the global specification of the GVAR model. It is straightforward to show that there exists a relationship between the prior variances on the weakly exogenous variables and the variances related to other countries' endogenous variables (termed global prior variances). As an illustration, consider  $\beta_{i1}^n$ , the coefficient associated with the first lag of the  $n$ th weakly exogenous variable, i.e.,  $x_{it}^{n*} = \sum_{j=0}^N \omega_{ij} x_{jt}^n$ , with prior variance given by  $\sigma_{in}^2$ . Then,  $\beta_{i1,j}^n = \omega_{ij} \beta_{i1}^n$  denotes country  $i$ 's coefficient corresponding to the  $n$ th variable of country  $j$  with (prior) variance given by  $\omega_{ij}^2 \sigma_{in}^2$ . Hence, the corresponding global prior variance is simply scaled down by the trade links between countries  $i$  and  $j$ .<sup>3</sup>

The Minnesota prior can be implemented by means of so-called dummy observations. Following Bańbura *et al.* (2010) and Koop (2013), the moments of the conjugate Minnesota prior can be matched attaching the following set of artificial dummy observations to the actual data

$$\underline{x}_i^M = \begin{pmatrix} 0_{1 \times k_i} \\ \dots \\ \text{diag}(\underline{a}_{i1} \sigma_{i1}, \dots, \underline{a}_{ik_i} \sigma_{ik_i}) / \alpha_{i1} \\ 0_{k_i(p-1) \times k_i} \\ \dots \\ 0_{k_i^*(p^*+1) \times k_i} \\ \dots \\ \text{diag}(\sigma_{i1}, \dots, \sigma_{ik_i}) \end{pmatrix}, \underline{Z}_i^M = \begin{pmatrix} \frac{1}{\alpha_3} & 0_{1 \times (K_i-1)} \\ \dots \\ 0_{k_i p \times 1} & J_p \otimes \text{diag}(\sigma_{i1}, \dots, \sigma_{ik_i}) / \alpha_1 & 0_{k_i p \times (K_i - k_i p - 1)} \\ \dots \\ 0_{(k_i^*(p^*+1)) \times (k_i p + 1)} & J_{p^*} \otimes \text{diag}(\sigma_{i1}^*, \dots, \sigma_{ik_i^*}^*) / \alpha_{i2} \\ \dots \\ 0_{k_i \times K_i} \end{pmatrix}, \quad (3.8)$$

where  $J_p = \text{diag}(1, 2, \dots, p)$  and  $J_{p^*} = \text{diag}(1, 2, \dots, p+1)$ . Additionally,  $0_{nq}$  denotes an  $n \times q$  dimensional matrix consisting exclusively of zeros.

The first block of  $\underline{x}_i$  and  $\underline{Z}_i$  implements the prior on the deterministic part of the model whereas the second block implements the random walk prior. Finally, the last two blocks implement the priors on the weakly exogenous variables and  $\Sigma_{\varepsilon i}$ , respectively.

Second, we consider the “sum-of-coefficients” prior, which softly forces the posterior distribution towards a specification in first differences. This implies that coefficients associated with own, lagged variables in each equation should sum to unity while other coefficients are being pushed towards zero. Implementation of this prior is straightforward by adding the following set of dummy observations to the data

$$\underline{x}_i^S = \left( \text{diag}(\underline{a}_{i1} \underline{\mu}_{i1}, \dots, \underline{a}_{ik_i} \underline{\mu}_{ik_i}) / \theta_{i1} \right) \\ \underline{Z}_i^S = \left( 0_{k_i \times 1} \quad \iota_{1 \times p} \otimes \text{diag}(\underline{a}_{i1} \underline{\mu}_{i1}, \dots, \underline{a}_{ik_i} \underline{\mu}_{ik_i}) / \theta_{i1} \quad 0_{k_i \times (k_i^*(p^*+1))} \right), \quad (3.9)$$

where  $\underline{\mu}_{ij}$  ( $j = 1, \dots, k_i$ ) denotes the pre-sample mean of the endogenous variables usually calculated by using the first  $p$  observations,  $\iota_{1 \times p}$  is a  $p$ -dimensional row vector of ones and  $\theta_{i1}$  is a country-specific hyperparameter controlling the tightness of the prior.

<sup>3</sup>A more formal treatment can be found in Appendix D

The fact that this prior is not consistent with cointegration gives rise to the “dummy-initial-observation” prior. This prior pushes variables in a country-specific VAR towards their unconditional (stationary) mean, or toward a situation where there is at least one unit root present. That is, either the process has a unit root, or it is stationary and starts near its mean, implying a penalty for models with inherent initial transient dynamics (Sims, 1992). Implementation boils down to attaching the following set of dummy observations to the actual data.

$$\begin{aligned}\underline{x}_i^I &= \left( \frac{(\mu_{i1}, \dots, \mu_{ik_i})}{\theta_{i2}} \right), \\ \underline{z}_i^I &= \begin{pmatrix} 0 & \iota_{1 \times p} \otimes \frac{(\mu_{i1}, \dots, \mu_{ik_i})}{\theta_{i2}} & \iota_{1 \times p^*} \otimes \frac{(\mu_{i1}^*, \dots, \mu_{ik_i}^*)}{\theta_{i2}} \end{pmatrix}.\end{aligned}\quad (3.10)$$

$\mu_{ij}^*$  ( $j = 1, \dots, k_i^*$ ) denote pre-sample averages from the weakly exogenous variables and  $\theta_{i2}$  is a hyperparameter controlling the tightness of the dummy-initial-observation prior.

In practice, macroeconomists usually incorporate all three versions of the random walk prior, which can be implemented in a straightforward fashion by combining the three pairs of dummy observations given in equations (3.8) to (3.10). The final prior, as motivated in Sims & Zha (1998), is then simply a weighted average of the three individual priors described above, where the weights attached to each prior are determined by the associated hyperparameters. Several studies have emphasized the usefulness of such a weighted prior structure (Bańbura *et al.*, 2010; Giannone *et al.*, 2013).

Natural conjugate priors require prior dependence between  $\Sigma_{\varepsilon i}$  and  $\Psi_i$ . The traditional implementation of the Minnesota prior drops this dependence, which provides more flexibility in terms of prior elicitation. In the empirical application in subsection 4.3 we also consider a non-conjugate variant of the Minnesota prior. This prior simply replaces the posterior of  $\Sigma_{\varepsilon i}$  by a known estimate  $\hat{\Sigma}_{\varepsilon i}$  which leads to analytical posterior solutions.

### Stochastic Search Variable Selection (SSVS) Prior

The conjugate priors discussed above apply by definition the same degree of shrinkage across equations. It might be appealing to provide more flexibility in the specification of the prior variance-covariance matrix on the coefficients and move away from the random walk prior model.

The SSVS prior, put forward by George & McCulloch (1993) and subsequently introduced to the VAR literature by George *et al.* (2008), imposes a mixture of Normal distributions on each coefficient of the VARX\*

$$\Psi_{ij} | \delta_{ij} \sim (1 - \delta_{ij}) \mathcal{N}(0, \tau_{0j}^2) + \delta_{ij} \mathcal{N}(\underline{\Psi}_{ij}, \tau_{1j}^2). \quad (3.11)$$

Here,  $\underline{\Psi}_{ij}$  denotes the prior mean and  $\delta_{ij}$  is a binary random variable corresponding to coefficient  $j$  in country model  $i$ . It equals one if the corresponding variable is included in the

model and zero if it is a priori excluded from the respective country model. The Normal distribution corresponding to  $\delta_{ij} = 0$  is typically specified with  $\tau_{0j}^2$  close to zero, which pushes the respective coefficient towards zero. The prior variance of the Normal distribution for  $\delta_{ij} = 1$ ,  $\tau_{1j}^2$ , is set to a comparatively large value implying a relatively uninformative prior on coefficient  $j$  conditional on inclusion. Note that, in contrast to the Minnesota prior, where the prior variance on the coefficients differs between different types of variables, the SSVS prior effectively applies an individual degree of shrinkage to every coefficient at the country level. The SSVS prior has been recently applied within a GVAR framework in [Feldkircher & Huber \(2015\)](#) to examine the international dimension of US economic shocks.

Defining a scalar parameter  $d_{ij}$  such that

$$d_{ij} = \begin{cases} \tau_{0j} & \text{if } \delta_{ij} = 1 \\ \tau_{1j} & \text{if } \delta_{ij} = 0 \end{cases} \quad (3.12)$$

and collecting all  $d_{ij}$  ( $j = 1, \dots, v_i$ ) in a  $v_i \times v_i$  matrix  $D_i = \text{diag}(d_{i1}, \dots, d_{iv_i})$  the prior on  $\Psi_i$  simply reduces to the following hierarchical prior setup

$$\Psi_i | D_i \sim \mathcal{N}(\underline{\Psi}_i, \underline{R}_i), \quad (3.13)$$

$$\Sigma_{\varepsilon i} \sim \mathcal{IW}(\underline{S}_i, \underline{v}_i), \quad (3.14)$$

where  $\underline{\Psi}_i$  equals a prior mean matrix,  $\underline{R}_i = D_i D_i$  and the prior on  $\Sigma_{\varepsilon i}$  is a standard inverse Wishart prior with prior degrees of freedom given by  $\underline{v}_i$  and prior scale matrix  $\underline{S}_i$ . Note that the lack of prior dependence between  $\Sigma_{\varepsilon i}$  and  $\Psi_i$  renders this prior (even conditionally) non-conjugate.

Finally, we follow [George \*et al.\* \(2008\)](#) and impose a Bernoulli prior on  $\delta_{ij}$

$$\delta_{ij} \sim \text{Bernoulli}(\underline{q}_{ij}), \quad (3.15)$$

where  $\underline{q}_{ij}$  denotes the prior inclusion probability of variable  $j$  in country  $i$ .

Estimation of this model requires Markov Chain Monte Carlo (MCMC) methods, although the conditional posteriors of  $\delta_{ij}$ ,  $\Psi_i$  and  $\Sigma_{\varepsilon i}$  are known. This implies that we can employ a simplified version of the Gibbs sampler outlined in [George \*et al.\* \(2008\)](#), where we start drawing  $\Psi_i$  from its full conditional posterior, which follows a Normal distribution. In the next step, we draw the latent variable  $\delta_{ij}$  from a Bernoulli distribution and in the last step we draw  $\Sigma_{\varepsilon i}$  from an inverse Wishart distribution.<sup>4</sup> This algorithm is repeated  $n$  times and the first  $n_{\text{burn}}$  draws are discarded as burn-ins. Averaging the draws of  $\delta_{ij}$  leads to posterior inclusion probabilities for each variable  $j$ . Further details are provided in [Appendix D](#).

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<sup>4</sup>In contrast to the implementation in [George \*et al.\* \(2008\)](#), we impose an inverse Wishart prior on  $\Sigma_{\varepsilon i}$  and depart from using a restriction search over  $\Sigma_{\varepsilon i}$ .

## 4 Empirical Results

### 4.1 Data and Model Specification

The bulk of the empirical literature employing GVARs use the dataset put forward in Dees *et al.* (2007b,a), which covers the most important economies in terms of real activity. For this dataset, time series are available from the early 1980s onward. Other studies have extended the country coverage to feature more emerging economies, at the price of limiting the available time span (Feldkircher, 2015; Feldkircher & Huber, 2015). In this paper, our aim is to reserve a significant share of our available time span for forecast evaluation, which is why we opt to have rather long time series – at the implied cost of reducing the country coverage.<sup>5</sup>

We rely on data provided in Dovern *et al.* (2015), that extend the dataset used in Dees *et al.* (2007a,b) with respect to variable coverage and time span. In what follows, we use quarterly data for 36 countries spanning the period from 1979:Q2 to 2013:Q4.<sup>6</sup>

The country-specific VARX\* models include seven domestic variables. Six variables are the same as in Dees *et al.* (2007a,b) and Pesaran *et al.* (2009), namely real GDP ( $y$ ), the change of the consumer price level ( $\Delta p$ ), real equity prices ( $eq$ ), the real exchange rate ( $e$ ) vis-à-vis the US dollar, and short-term ( $i_s$ ) and long-term interest rates ( $i_l$ ). We enlarge this set of variables to feature total credit ( $tc$ , domestic and cross-border credit), as a seventh financial variable. This seems to be important, as the hold-out sample for our forecasting exercise contains the global financial crisis, which spread via both the trade and the financial channel.<sup>7</sup> Note that not all variables are available for each of the countries we consider in this study. With the exception of long-term interest rates, the cross-country coverage of all variables is, however, above 80%. Long-term interest rate data are missing for emerging markets that are characterized by underdeveloped capital markets.

The vector of domestic variables for a typical country  $i$  is thus given by

$$\mathbf{x}_{it} = (y_{it}, \Delta p_{it}, e_{it}, eq_{it}, i_{sit}, i_{lit}, tc_{it})'. \quad (4.1)$$

We follow the bulk of the literature by including oil prices ( $poil_t$ ) as a global control variable. With the exception of the bilateral real exchange rate, we construct foreign counterparts

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<sup>5</sup>Note that this is in contrast to the working paper version of this study available at <http://www.oenb.at/Publikationen/Volkswirtschaft/Working-Papers/2014/Working-Paper-189.html> which also includes a forecast comparison to the standard, cointegrated GVAR model put forward in Pesaran *et al.* (2004) and Dees *et al.* (2007a,b), among others.

<sup>6</sup>The following countries are included in the respective regions: Europe includes Austria (AT), Belgium (BE), Germany (DE), Spain (ES), Finland (FI), France (FR), Greece (GR), Italy (IT), Netherlands (ND), Portugal (PT), Denmark (DK), Great Britain (GB), Switzerland (CH), Norway (NO) and Sweden (SE). Other Developed economies feature Australia (AU), Canada (CA), Japan (JP), New Zealand (NZ) and the USA (US). Emerging Asia includes China (CN), India (IN), Indonesia (ID), Malaysia (MY), Korea (KR), Philippines (PH), Singapore (SG) and Thailand (TH). Latin America comprises Argentina (AR), Brazil (BR), Chile (CL), Mexico (MX) and Peru (PE). Mid-East and Africa consists of Turkey (TR), Saudi Arabia (SA) and South Africa (ZA).

<sup>7</sup>For a more detailed description, see Table A.1 in Appendix A.

for all domestic variables. The weights to calculate foreign variables are based on average bilateral annual trade flows in the period from 1980 to 2003, which denotes the end of our initial estimation sample.<sup>8</sup> For a typical country  $i$  the set of weakly exogenous and global control variables comprises the following variables,

$$\mathbf{x}_{it}^* = (y_{it}^*, \Delta p_{it}^*, eq_{it}^*, i_{sit}^*, i_{lit}^*, tc_{it}^*, poil_t^*)'. \quad (4.2)$$

The US model ( $i = 0$ ) deviates from the other country specifications in that the oil price ( $poil_t$ ) is determined within that country model and the trade weighted real exchange rate ( $e_t^*$ ) is included to control for co-movements of currencies,

$$\mathbf{x}_{0t} = (y_{0t}, \Delta p_{0t}, eq_{0t}, i_{s0t}, i_{l0t}, tc_{0t}, poil_t)'. \quad (4.3)$$

$$\mathbf{x}_{0t}^* = (y_{0t}^*, \Delta p_{0t}^*, e_{0t}^*, eq_{0t}^*, i_{s0t}^*, i_{l0t}^*, tc_{0t}^*)'. \quad (4.4)$$

The dominant role of the US economy for global financial markets is often accounted for by including a limited set of weakly exogenous variables in its country-specific model. In our setting, coefficients attached to all variables are subject to the shrinkage induced by the priors reviewed in the previous section. It is therefore a priori reasonable to include a large set of international macroeconomic and financial indicators to avoid ad-hoc restrictions, even for a large country such as the US.

For all countries considered, we set the lag length of endogenous and weakly exogenous variables equal to five. Given the quarterly frequency of the data and the fact that we introduce Bayesian shrinkage, this seems to be a reasonable choice. We correct for outliers in countries that witnessed extraordinarily strong crisis-induced movements in some of the variables contained in our data. We opted to smooth the relevant time series in these cases rather than include step dummies. While step dummies might control for outliers within the specific country model, extreme shocks might still be carried over to other country models via trade-weighted foreign variables. Obviously, this is not the case when smoothing the series in the first place.<sup>9</sup>

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<sup>8</sup>Note that recent contributions (Eickmeier & Ng, 2015; Doornik & van Roye, 2014) suggest using financial data to compute foreign variables related to the financial side of the economy (e.g., interest rates or credit volumes). Since our data sample starts in the early 1980s, reliable data on financial flows – such as portfolio flows or foreign direct investment – are not available. See Feldkircher & Huber (2015) for a sensitivity analysis with respect to the choice of weights.

<sup>9</sup>We define outliers as those observations that exceed 1.5 times the interquartile range in absolute value. The identified outliers are then smoothed using cubic spline interpolation techniques and in case they are located at the tails of the sample – extrapolation techniques. Using the definition of the interquartile range, we identify 2% of our sample as unusual observations. From these 2%, about 60% regard unusual observations for inflation at the beginning of the observation sample. Short-term interest rates (20%) and the real exchange rate (13%) have historically also shown very volatile patterns for the countries covered in this study. More detailed country-specific information is available from the authors upon request.

## 4.2 Selection of Hyperparameters

Due to the strong heterogeneity observed in the world economy, it is daunting to assume that different countries obey the same structural dynamics in terms of macroeconomic fundamentals. Thus, using the same set of hyperparameters when eliciting the prior for all countries considered might be too restrictive to unveil differences between economies.

The conjugate priors rely on a set of (presumably fixed) hyperparameters which are homogeneous across countries. A specific set of hyperparameters could, however, induce a tight prior in one country while being relatively loose in other countries. To avoid this problem, we follow [Carriero \*et al.\* \(2015\)](#) and choose the hyperparameters by maximizing the marginal likelihood on a discrete grid of values for  $\underline{\alpha}_1$  and  $\underline{\alpha}_2$ . For the remaining parameters, we set  $\underline{\alpha}_{i3} = 100$  and  $\theta_{i1} = \theta_{i2} = 1$ . For the natural conjugate prior, the marginal likelihood is available in closed form (see, for instance, [Bauwens \*et al.\*, 2000](#); [Koop, 2013](#)). Using the marginal likelihood as a loss function is motivated by the fact that it can be written as a sequence of one-step-ahead predictive densities. Thus maximizing the marginal likelihood under a flat prior is equivalent to minimizing the one-step-ahead prediction errors ([Geweke, 2001](#); [Geweke & Whiteman, 2006](#)).<sup>10</sup> Since the marginal likelihood is not available in closed form for the standard Minnesota prior, we use the hyperparameters obtained from the conjugate prior for this setting. Following [Carriero \*et al.\* \(2015\)](#), the parameter controlling the degree of shrinkage on “other” variables is set equal to 0.8. Finally, we set  $\underline{a}_{ij}$  equal to unity for the first lag of “own” variables except for inflation, where we set  $\underline{a}_{ij} = 0.2$ . This choice, stipulated in [Clark \(2011\)](#), is consistent with the notion that all variables are non-stationary.<sup>11</sup>

For the SSVS prior, we set the prior inclusion probability for each variable equal to 0.5, which implies that a priori, every variable is assumed equally likely to enter the model. We set  $\tau_{i,0j} = 0.1s_{ij}$  and  $\tau_{i,1j} = 10s_{ij}$  and rely on the semi-automatic approach described in [George \*et al.\* \(2008\)](#) to scale the hyperparameters, where  $s_{ij}$  is the standard error attached to coefficient  $j$  based on a VARX\* estimated by OLS in country  $i$ . Finally, we set  $\underline{S}_i = 10I_{k_i}$ , the prior degrees of freedom  $\underline{v}_i$  to  $k_i$  and  $\underline{\Psi}_i$  equal to the zero matrix. As a robustness check we also used a standard random walk prior specification in combination with the SSVS prior. This implies that the prior mean on the first own lag is set equal to unity. However, since almost no shrinkage is imposed for the case when  $\delta_{ij} = 1$  the results are rather similar with the standard SSVS implementation. Thus for the sake of brevity we only report the results obtained by setting the prior mean equal to zero for all coefficients. To assess the importance of shrinkage we include in the forecast exercise a prior that is flat over the coefficients (diffuse).

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<sup>10</sup>Note that this differs from the procedure proposed by [Giannone \*et al.\* \(2012\)](#), since we do not integrate out the hyperparameters in a Bayesian fashion but simply plug in an estimate of the posterior mode of  $\underline{\alpha}_1$  and  $\underline{\alpha}_2$  under a diffuse prior. This approach seems convenient since it avoids MCMC sampling for the conjugate priors, which proves to be important for the empirical application.

<sup>11</sup> Setting the prior mean for the first lag of inflation equal to unity leads to qualitatively similar results.

This prior is implemented by setting  $\underline{\alpha}_{i1} = \underline{\alpha}_{i2} = \underline{\alpha}_{i3} = 10^{10}$  in the conjugate Minnesota prior setup.

### 4.3 Forecast Performance

The initial estimation period ranges from 1979:Q2 to 2003:Q4 and we use the period 2004:Q1-2013:Q4 as out-of-sample hold-out observations to compare predictive performance across specifications. We base our comparison on recursive one-quarter-ahead and four-quarter-ahead predictions obtained by re-estimating the models over an expanding window defined by the beginning of the available sample and the corresponding period in the hold-out sample. In what follows, we compare the forecast performance for both point and density forecasts. For means of comparison we choose the root mean squared error (RMSE) for point forecasts and log predictive scores (LPS) to evaluate density forecasts.<sup>12</sup> All forecasts are benchmarked to those of a fifth-order univariate autoregressive (AR(5)) model and forecast errors are reported in an unweighted fashion. That is, we do not attach more weight to favor GVAR specifications that improve forecasts for particular countries which stand out in terms of economic activity. The AR(5) model is estimated in a Bayesian fashion by setting the prior variances of the non-conjugate Minnesota prior such that almost no shrinkage is imposed on the coefficients associated with a given variable while all other coefficients are strongly pushed towards zero.

#### Forecast Evaluation: Overall Results

Overall results based on one-step-ahead forecasts are provided in the upper panel of [Table 1](#). Considering results on point forecasts first, most specifications yield forecasts that improve upon the naive model, as indicated by relative RMSEs below unity.

We start by investigating the relative merits of using shrinkage priors on the one hand, and a global model (without shrinkage on the coefficients) on the other hand. Comparing the results of the BVAR specification with that of the GVAR under a diffuse prior does not yield clear-cut results concerning the superiority of one of the two modelling frameworks for all variables considered. For five out of seven variables, both settings yield on average point forecasts that are more accurate than those of the naive model. Combining the virtues of both approaches (shrinkage and international linkages), however, boosts forecast performance considerably. All B-GVAR prior settings that induce shrinkage outperform the benchmark model. Only one-step-ahead forecasts for inflation under the SSVS prior are similar to those of the benchmark in terms of RMSEs. Comparing B-GVAR shrinkage specifications, our results show a particular good forecast performance for the non-conjugate Minnesota (M-NC) prior and the prior proposed by [Sims & Zha \(1998\)](#) (S&Z). The non-conjugate Minnesota prior yields the best point forecasts for four, and the S&Z prior for two out of the seven variables

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<sup>12</sup>See [Appendix E](#) for more details on the construction of the density measures.



considered. The forecast performance of the conjugate variant of the Minnesota prior (M-C) is less spectacular, yielding prediction accuracy measures close to that of the B-GVAR under a diffuse prior setting. Finally, the SSVS framework excels in forecasting total credit, while for the remaining variables forecasts are slightly worse than the ones of the M-NC and S&Z specifications.

Next, we evaluate the relative quality of density forecasts. Table 1, upper panel right-hand side, displays the sum of log predictive scores over countries per variable, reported as differences to the benchmark autoregressive model. Positive values indicate a better performance of the forecast method under consideration compared to the benchmark. The results confirm the findings for point forecast accuracy. Both isolated country-specific VAR models (BVAR) and the international model without shrinkage (diffuse) outperform forecasts of the naive model. Again, there is no clear superiority structure when comparing these two approaches, while forecasts can be further improved by considering models that explicitly feature international linkages coupled with priors that induce shrinkage on the parameters (M-C, S&Z, M-NC and SSVS). Improvements over forecasts from the benchmark are most pronounced for the SSVS prior and the M-NC specification. The S&Z and M-C priors also yield forecast gains compared to the benchmark but these are smaller than under the SSVS and M-NC models.

Finally, we consider results based on a four-quarters-ahead forecast horizon. In the medium-term, forecasts from the naive benchmark specification appear hard to beat. The merits from using shrinkage priors play out more strongly in this setting than in the short-term prediction exercise. While forecasts from the isolated country-specific BVAR models do not tend to worsen markedly with the expanded forecast horizon, the GVAR model under a diffuse prior setting shows a poor forecasting performance. This holds true for both point and density forecasts. In line with our findings on the one-step-ahead forecast horizon, forecasts can be further improved by considering GVAR specifications coupled with shrinkage priors. However, forecast gains are less pronounced than in the short run. Among the prior specifications considered, the S&Z prior excels in point forecast accuracy, while the SSVS and the M-NC prior yield the strongest performance regarding density forecasts.

[Table 1 about here.]

### **Forecast Evaluation: Cross-Country Differences in Point Forecast Accuracy**

In order to examine whether there are systematic cross-country differences in point forecast accuracy for the priors considered, Figures 1 to 5 show the cross-sectional distribution of relative RMSE for the one-step-ahead forecast horizon for different world regions. We present



results for Europe, other developed economies, emerging Asia and Latin America.<sup>13</sup> With the exception of Latin America, all plots have the same scaling in order to ease regional comparison of forecast accuracy under the different specifications.

[Figure 1 about here.]

[Figure 2 about here.]

[Figure 3 about here.]

[Figure 4 about here.]

[Figure 5 about here.]

The results indicate that point forecast accuracy varies strongly across regions and less so across variables. Taking a regional stance, the largest dispersion of relative RMSE values is observed for variables in Latin American economies. Here the cross-sectional variance of forecast accuracy tends to be large for practically all prior specifications considered. Point forecasts are particularly inaccurate and cross-sectional distributions wide for inflation, total credit and short-term interest rates. For these variables the distributions are about three to four times larger than the ones for the rest of the regions and medians of the distribution of RMSE tend to indicate a worse performance relative to the naive model. Considering the distribution of point forecast accuracy within Latin America, all specifications indicate real exchange rates for Argentina to be particularly hard to forecast. While the dispersion of relative RMSE is markedly smaller in emerging Asia, some of the priors considered yield very inaccurate point forecasts for real activity and inflation in India and Indonesia. By comparison, forecast performance is very homogeneous for variables of European and other developed economies. The distributions of relative predictive ability tend to be very tight, and forecasts that fall far off the median do so for most priors considered. Inaccurate point forecasts for real GDP can be found for Norway, whose economy depends strongly on oil exports, Denmark, Japan and New Zealand. For total credit and the real exchange rate, relative RMSE figures are particularly pronounced in Great Britain and Switzerland, two countries with a large financial sector. Countries that appear as outliers in the box plots might either indicate that some country-specific features (e.g., oil based economy, heavy financial sector) are not correctly reflected in the model, the (linear) specification of the model might be too restrictive or the variance of the underlying time series might be comparably large.

Comparing across prior specifications, these disaggregated results corroborate the findings provided in [Table 1](#). Concentrating on the median value, most prior specifications tend to

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<sup>13</sup>We consider all variables but long-term interest rates and equity prices, for which the cross-country coverage is limited. Results for these variables as well as for the four-quarters-ahead forecast horizon are available upon request.

outperform forecasts from the naive models for all regions and variables with the exception of Latin America. Forecasts under the M-NC, S&Z and SSVS priors often yield the lowest cross-sectional median of relative RMSEs. At first sight, also the diffuse prior yields quite accurate point forecasts. However, the distribution of relative RMSEs tend to be tighter when using priors that incorporate country-specific shrinkage. This holds in particular true for Latin American variables, where relative RMSE distributions under some shrinkage priors are very tight, while the forecast performance indicator is very disperse under the diffuse prior.

## 5 Conclusions

In this paper we develop a Bayesian GVAR model and assess its out-of-sample predictive performance in terms of point- and density forecasts. We use a large quarterly dataset that starts in 1979:Q2, excels in country coverage and covers many of the most important macroeconomic and financial variables. This dataset allows us to reserve a significant share of the data as a forecast evaluation sample (40 observations by country, spanning the period from 2004:Q1 to 2013:Q4). Our forecast evaluation sample thus includes periods of very distinct macroeconomic and financial conditions: the period of the great moderation that was accompanied by stable GDP growth and low inflation, followed by the global financial crisis which triggered most of the economies to enter (prolonged) recession phases, and the ongoing period of recovery since then. Evaluating forecasts over that period yields a fair assessment of the usefulness of Bayesian GVAR models, and the length of our hold-out sample significantly improves upon earlier studies on forecasting using GVAR specifications (see, e.g., [Pesaran \*et al.\*, 2009](#)).

Our *main results* are the following. First, we provide ample evidence that taking *international linkages* among the economies into account and using priors that *induce shrinkage on the parameters locally* greatly improves forecast performance. Throughout the set of variables considered in this study, the diffuse prior setup that is flat over the coefficients as well as forecasts from isolated Bayesian VAR models do not tend to rank among the best performing forecast specifications. This holds true for both point forecasts and density forecasts as well as the short-term and medium-term horizon. To set the degree of shrinkage for each country model locally, we numerically optimize the marginal likelihood with respect to the hyperparameters, which is equivalent to minimizing the one-step-ahead prediction errors. This allows us to accommodate a large degree of heterogeneity across the economies, which appears of particular importance for forecasting in a global setting. Second, within the class of Bayesian GVARs that induce country-specific shrinkage, *no single prior dominates all forecasting setups*. The S&Z, the non-conjugate version of the Minnesota prior (M-NC) and the SSVS prior all show a very strong forecast performance over the hold-out sample, while the forecast improvement of the conjugate Minnesota prior compared to the naive benchmark is small.

The non-conjugate Minnesota prior shows an excellent track record of predictive ability for the one-step-ahead forecast horizon in terms of point forecasts and together with the SSVS prior it excels in short-term density forecasts. The latter also shows a strong density forecast performance in the longer run. The S&Z prior excels in point forecast accuracy and in particular so in the longer run, where its flexibility by that induces shrinkage in various dimensions (including soft-differencing of the data) seems to pay off. Third, *taking a regional stance*, our results indicate that forecasts for Latin America are particularly inaccurate for most specifications considered in this study. By contrast, forecast performance for variables from European or other developed economies is much more homogeneous and differences between the various prior setups considered more modest.

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**Table 1:** Forecast performance relative to autoregressive benchmark

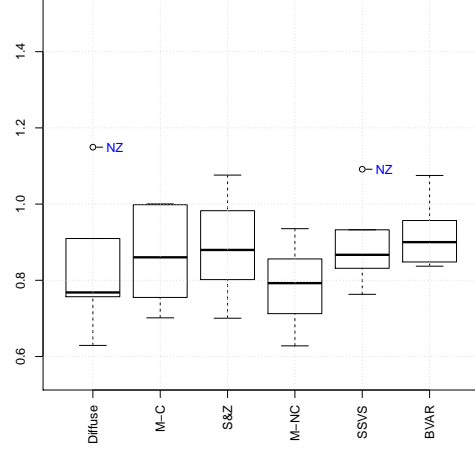
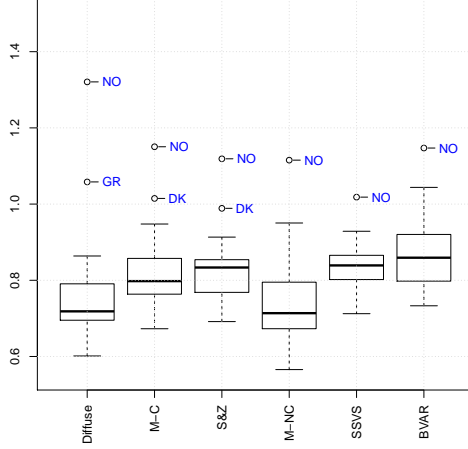
	RMSE										LPS		
	One-quarter-ahead										Non-Conjugate		
	Conjugate		Non-Conjugate		Diffuse	BVAR	Conjugate		Non-Conjugate		M-NC	SSVS	BVAR
	M-C	S&Z	M-NC	SSVS			M-C	S&Z	M-NC	SSVS			
$y$	0.846	0.900	0.909	0.883	0.942	0.942	179.872	183.131	136.815	<b>214.057</b>	136.815	<b>214.057</b>	175.001
$\Delta p$	1.019	0.911	<b>0.901</b>	0.963	0.930	0.930	12.053	13.398	<b>14.616</b>	0.157	<b>14.616</b>	0.157	9.292
$e$	0.964	0.978	1.002	0.949	1.009	1.009	35.546	<b>35.563</b>	30.076	26.046	30.076	26.046	35.373
$i_S$	1.065	0.839	<b>0.778</b>	0.877	0.833	0.833	4.790	5.665	<b>12.111</b>	5.171	<b>12.111</b>	5.171	2.207
$i_L$	0.830	0.835	0.837	0.875	0.863	0.863	6.843	5.850	<b>9.823</b>	-2.194	<b>9.823</b>	-2.194	2.936
$eq$	0.888	0.857	0.869	0.826	0.877	0.877	263.597	269.999	270.763	<b>276.233</b>	270.763	<b>276.233</b>	266.618
$tc$	1.036	0.969	0.998	<b>0.915</b>	1.037	1.037	194.628	196.012	120.955	<b>265.352</b>	120.955	<b>265.352</b>	224.532
	Four-quarters-ahead										Non-Conjugate		
	Conjugate		Non-Conjugate		Diffuse	BVAR	Conjugate		Non-Conjugate		M-NC	SSVS	BVAR
	M-C	S&Z	M-NC	SSVS			M-C	S&Z	M-NC	SSVS			
$y$	1.788	0.935	<b>0.865</b>	1.149	1.046	0.895	135.005	137.740	101.006	<b>150.384</b>	101.006	<b>150.384</b>	131.648
$\Delta p$	1.883	1.053	<b>0.977</b>	1.405	1.138	1.025	1.549	<b>2.420</b>	-4.614	-12.676	-4.614	-12.676	0.241
$e$	2.234	1.204	1.105	1.723	<b>1.066</b>	1.094	12.057	15.255	9.611	<b>19.700</b>	9.611	<b>19.700</b>	17.221
$i_S$	2.090	1.095	<b>0.939</b>	1.308	1.096	0.976	3.927	4.970	<b>10.646</b>	7.102	<b>10.646</b>	7.102	0.692
$i_L$	1.864	1.011	<b>0.979</b>	1.163	0.987	0.985	5.158	3.897	<b>5.352</b>	-3.578	<b>5.352</b>	-3.578	-0.199
$eq$	3.080	1.101	1.029	1.831	1.293	<b>0.987</b>	172.428	173.119	164.492	<b>175.140</b>	164.492	<b>175.140</b>	170.053
$tc$	1.235	0.707	<b>0.680</b>	0.881	0.696	0.685	190.414	194.068	135.085	<b>234.829</b>	135.085	<b>234.829</b>	212.790

Notes: The figures on the left panel refer to root mean squared error (RMSE) relative to an autoregressive model of order five (AR(5)). Values below one indicate better performance of the respective forecast model compared to the benchmark. Figures on the right panel refer to the differences of the sum of variable specific log predictive scores (LPS) relative to the AR(5) benchmark. Results based on forecasts over the time period 2004:Q1-2013:Q4. Figures for best performing models are in bold. *Diffuse* stands for the model estimated using maximum likelihood, *M-C* denotes the GVAR with the conjugate variant of the Minnesota prior, *S&Z* refers to a GVAR estimated using a weighted average of the conjugate priors as in Sims & Zha (1998), *M-NC* stands for the non-conjugate variant of the Minnesota prior, *SSVS* denotes the GVAR estimated using the SSVS prior and *BVAR* denotes a set of isolated, country-specific vector autoregressions estimated using the S&Z prior.

**Figure 1:** Cross-sectional distribution of 1-step-ahead RMSE values for real GDP

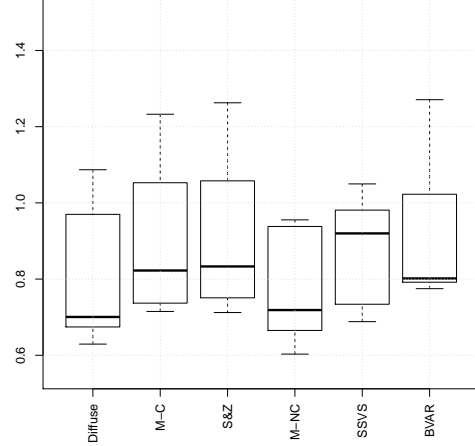
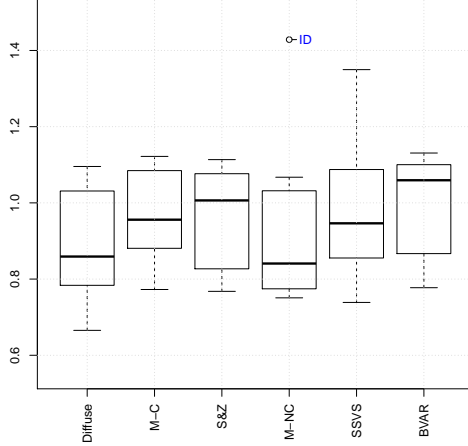
(a) Europe

(b) Other developed economies



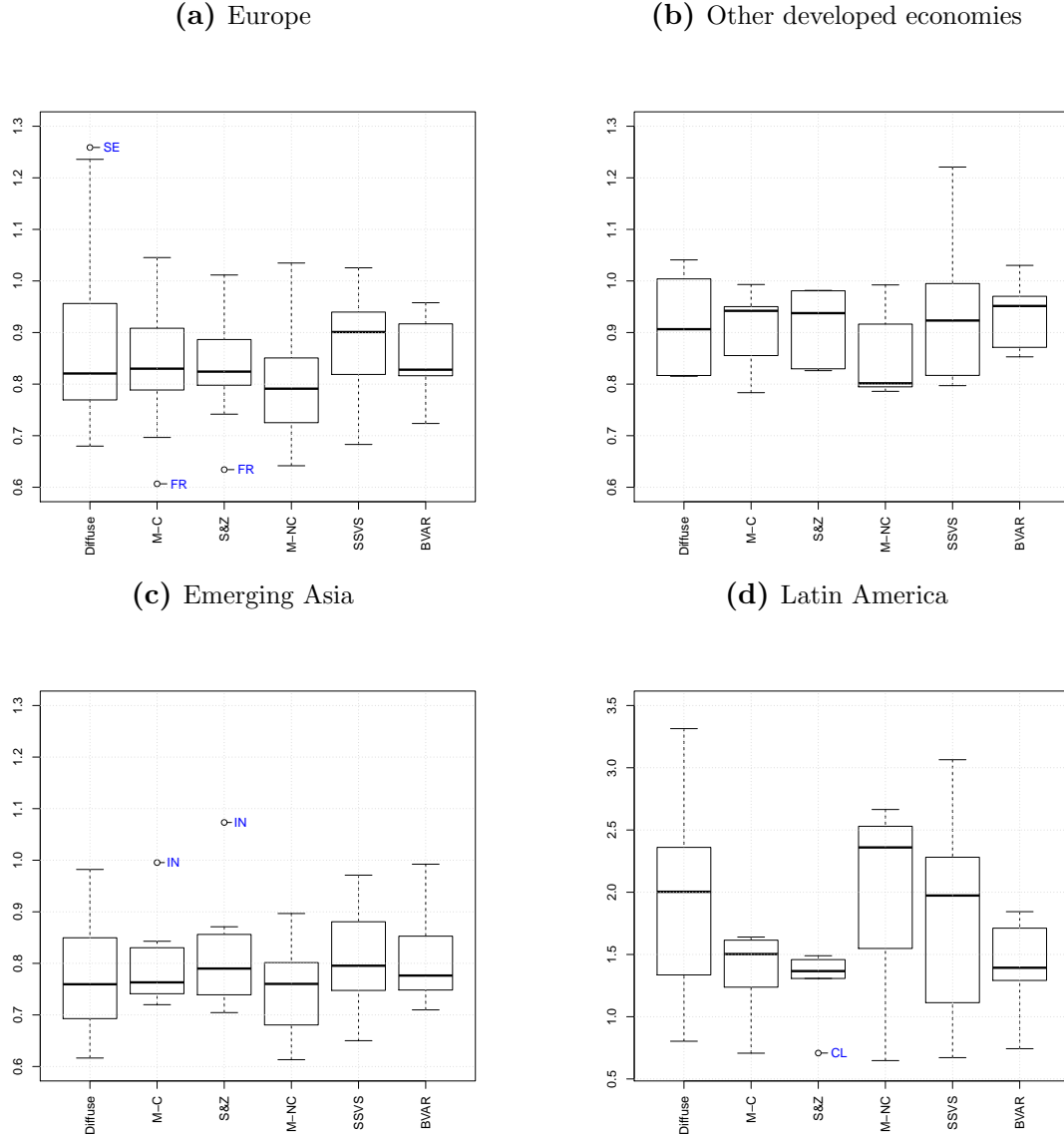
(c) Emerging Asia

(d) Latin America



**Notes:** The figures show the cross-sectional distribution of the ratio of the RMSE corresponding to the model to the RMSE of an autoregressive model of order five over the time period 2004:Q1-2013:Q4. Diffuse stands for the model estimated using maximum likelihood, M-C denotes the GVAR with the conjugate variant of the Minnesota prior, S&Z refers to a GVAR estimated using a weighted average of the conjugate priors as in [Sims & Zha \(1998\)](#), M-NC stands for the non-conjugate variant of the Minnesota prior, SSVS denotes the GVAR estimated using the SSVS prior, and BVAR denotes forecasts based on separate country VARs with the S&Z prior employed. Observations that exceed 1.5 times the interquartile range are marked as outliers.

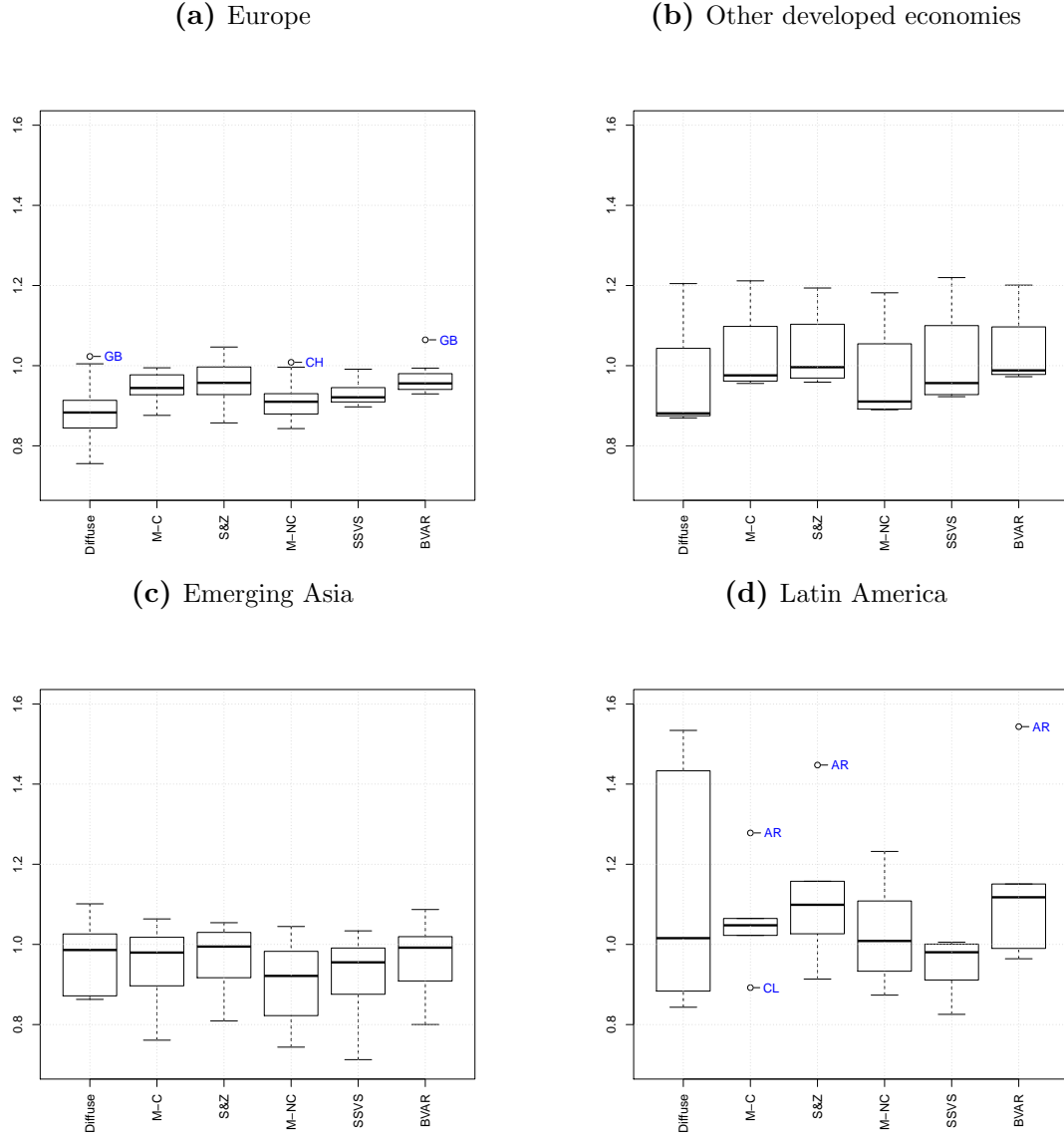
**Figure 2:** Cross-sectional distribution of 1-step-ahead RMSE values for inflation



**Notes:** The figures show the cross-sectional distribution of the ratio of the RMSE corresponding to the model to the RMSE of an autoregressive model of order five over the time period 2004:Q1-2013:Q4. Diffuse stands for the model estimated using maximum likelihood, M-C denotes the GVAR with the conjugate variant of the Minnesota prior, S&Z refers to a GVAR estimated using a weighted average of the conjugate priors as in [Sims & Zha \(1998\)](#), M-NC stands for the non-conjugate variant of the Minnesota prior, SSVS denotes the GVAR estimated using the SSVS prior, and BVAR denotes forecasts based on separate country VARs with the S&Z prior employed. Observations that exceed 1.5 times the interquartile range are marked as outliers.

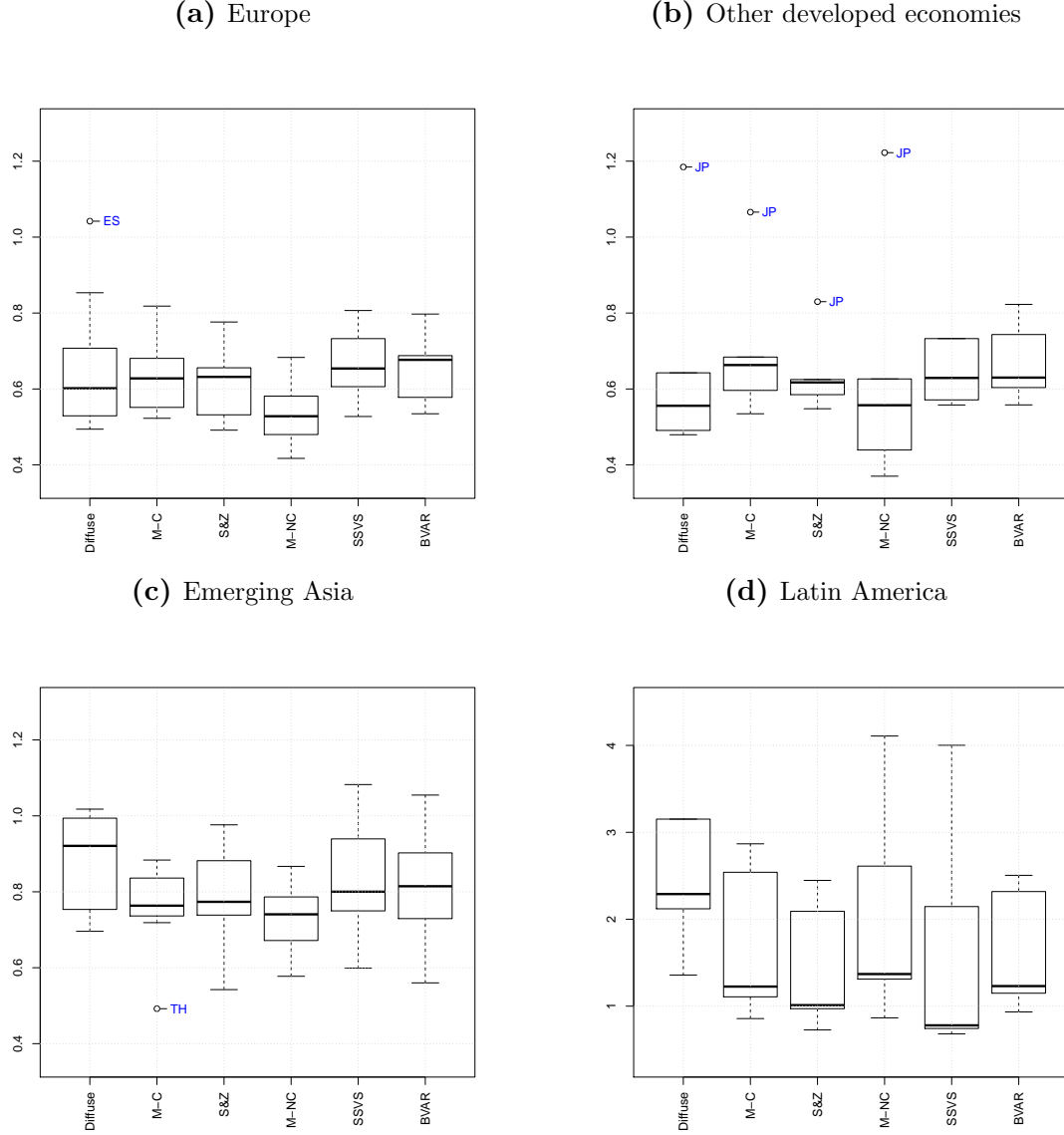


**Figure 3:** Cross-sectional distribution of 1-step-ahead RMSE values for the real exchange rate



**Notes:** The figures show the cross-sectional distribution of the ratio of the RMSE corresponding to the model to the RMSE of an autoregressive model of order five over the time period 2004:Q1-2013:Q4. Diffuse stands for the model estimated using maximum likelihood, M-C denotes the GVAR with the conjugate variant of the Minnesota prior, S&Z refers to a GVAR estimated using a weighted average of the conjugate priors as in [Sims & Zha \(1998\)](#), M-NC stands for the non-conjugate variant of the Minnesota prior, SSVS denotes the GVAR estimated using the SSVS prior, and BVAR denotes forecasts based on separate country VARs with the S&Z prior employed. Observations that exceed 1.5 times the interquartile range are marked as outliers.

**Figure 4:** Cross-sectional distribution of 1-step ahead RMSE values for short-term interest rates

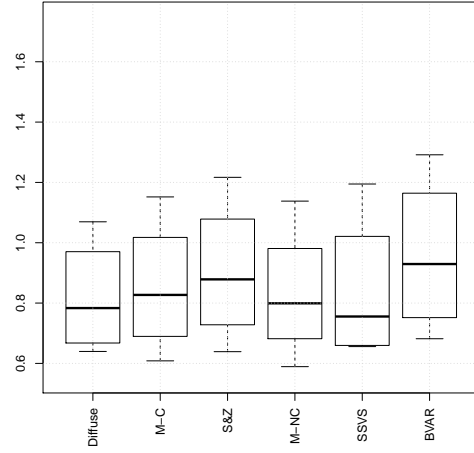
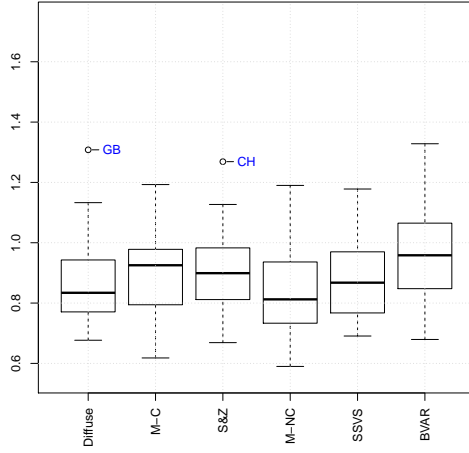


**Notes:** The figures show the cross-sectional distribution of the ratio of the RMSE corresponding to the model to the RMSE of an autoregressive model of order five over the time period 2004:Q1-2013:Q4. Diffuse stands for the model estimated using maximum likelihood, M-C denotes the GVAR with the conjugate variant of the Minnesota prior, S&Z refers to a GVAR estimated using a weighted average of the conjugate priors as in [Sims & Zha \(1998\)](#), M-NC stands for the non-conjugate variant of the Minnesota prior, SSVS denotes the GVAR estimated using the SSVS prior, and BVAR denotes forecasts based on separate country VARs with the S&Z prior employed. Observations that exceed 1.5 times the interquartile range are marked as outliers.

**Figure 5:** Cross-sectional distribution of 1-step-ahead RMSE values for total credit

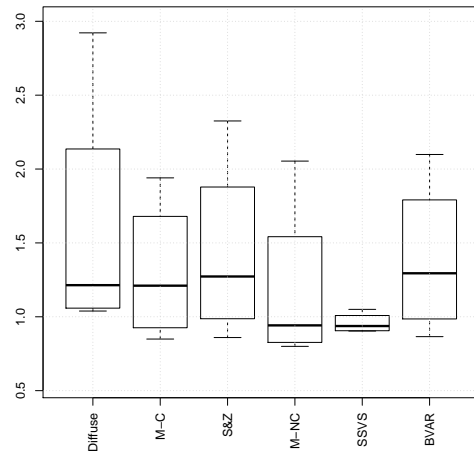
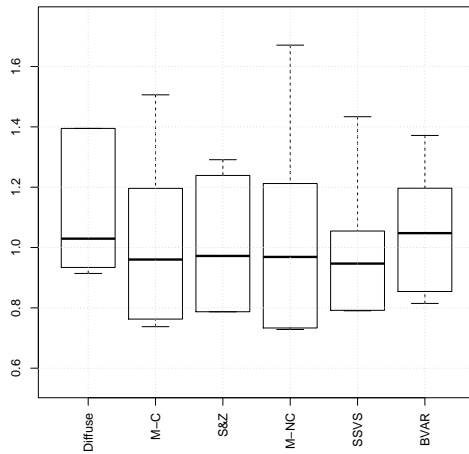
(a) Europe

(b) Other developed economies



(c) Emerging Asia

(d) Latin America



**Notes:** The figures show the cross-sectional distribution of the ratio of the RMSE corresponding to the model to the RMSE of an autoregressive model of order five over the time period 2004:Q1-2013:Q4. *Diffuse* stands for the model estimated using maximum likelihood, *M-C* denotes the GVAR with the conjugate variant of the Minnesota prior, *S&Z* refers to a GVAR estimated using a weighted average of the conjugate priors as in [Sims & Zha \(1998\)](#), *M-NC* stands for the non-conjugate variant of the Minnesota prior, *SSVS* denotes the GVAR estimated using the SSVS prior and *BVAR* denotes a set of isolated, country-specific vector autoregressions estimated using the S&Z prior. Observations that exceed 1.5 times the interquartile range are marked as outliers.

## Appendix A Data Description

**Table A.1:** Data description

Variable	Description	Min.	Mean	Max.	Coverage
$y$	Real GDP, average of 2005=100. Seasonally adjusted, in logarithms.	2.173	4.298	5.400	100%
$\Delta p$	Consumer price inflation. CPI seasonally adjusted, in logarithms.	-0.157	0.021	0.660	100%
$e$	Nominal exchange rate vis-à-vis the US dollar, deflated by national price levels (CPI).	-5.373	-2.814	4.968	97.2%
$i_S$	Typically 3-months-market rates, rates per annum.	-0.001	0.118	5.189	97.2%
$i_L$	Typically government bond yields, rates per annum.	0.000	0.077	0.275	61.1%
$tc$	Total credit (domestic + cross border), seasonally adjusted, in logarithms, average of 2005=100.	-14.140	3.514	6.552	83.33%
$poil$	Price of oil, seasonally adjusted, in logarithms.	-	-	-	-
Trade flows	Bilateral data on exports and imports of goods and services, annual data.	-	-	-	-

**Notes:** Summary statistics pooled over countries and time. The coverage refers to the cross-country availability per country, in %. Data are from the IMF's IFS data base and national sources. Trade flows stem from the IMF's DOTS data base. For more details see the data appendix in [Feldkircher \(2015\)](#).

## Appendix B Deriving the GVAR Model

For the sake of exposition, let us assume that  $p = 1$ ,  $p^* = 1$  and  $a_{i0} = 0$ . Following [Pesaran et al. \(2004\)](#), the country-specific models in equation (2.1) can be rewritten as

$$A_i z_{it} = B_i z_{it-1} + \varepsilon_{it}, \quad (\text{B.1})$$

where  $A_i = (I_{k_i}, -\Lambda_{i0})$ ,  $B_i = (\Phi_i, -\Lambda_{i1})$  and  $z_{it} = (x'_{it}, x'^*_{it})'$ . By defining a suitable link matrix  $W_i$  of dimension  $(k_i + k_i^*) \times k$ , where  $k = \sum_{i=0}^N k_i$ , we can rewrite  $z_{it}$  as  $z_{it} = W_i x_t$ , with  $x_t$  (the so-called global vector) being a vector where all the endogenous variables of the countries in our sample are stacked, i.e.,  $x_t = (x'_{0t}, \dots, x'_{Nt})'$ . Replacing  $z_{it}$  with  $W_i x_t$  in (B.1) and stacking the different local models leads yields the global model,

$$\begin{aligned} x_t &= G^{-1} H x_{t-1} + G^{-1} \epsilon_t \\ &= F x_{t-1} + e_t. \end{aligned} \quad (\text{B.2})$$

Here,  $G = ((A_0 W_0)', \dots, (A_N W_N'))'$  and  $H = ((B_0 W_0)', \dots, (B_N W_N'))'$  denote the corresponding stacked matrices containing the parameter matrices of the country-specific speci-

cations. In line with existing work (e.g., [Dees et al., 2007b](#)) we assume that  $G$  is invertible. Finally,  $e_t \sim \mathcal{N}(0, \Sigma_e)$ , where  $\Sigma_e = G^{-1}\Sigma_\epsilon(G^{-1})'$  and  $\Sigma_\epsilon$  is a block-diagonal matrix given by

$$\Sigma_\epsilon = \begin{pmatrix} \Sigma_{\epsilon 0} & 0 & \cdots & 0 \\ 0 & \Sigma_{\epsilon 1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Sigma_{\epsilon N} \end{pmatrix}. \quad (\text{B.3})$$

Consequently, the matrix  $G$  establishes contemporaneous cross-country correlations. The eigenvalues of the matrix  $F$  provide information about the stability of the global system. In the empirical application we rule out explosive behavior of the model by discarding posterior draws that significantly fall outside the unit circle. The framework outlined above deviates from the work pioneered by [Pesaran et al. \(2004\)](#) in that we do not explicitly impose cointegration relationships in the individual country-specific models.

## Appendix C Posterior Distributions

### The Conjugate Case

For all priors discussed in [Section 3](#) that can be cast into a form that uses dummy observations, prior quantities can be expressed as

$$\underline{\Pi}_i = (\underline{Z}_i' \underline{Z}_i)^{-1} \underline{Z}_i' \underline{x}_i \quad (\text{C.1})$$

$$\underline{V}_i = (\underline{Z}_i' \underline{Z}_i)^{-1} \quad (\text{C.2})$$

$$\underline{S}_i = (\underline{x}_i - \underline{Z}_i \underline{\Pi}_i)' (\underline{x}_i - \underline{Z}_i \underline{\Pi}_i) \quad (\text{C.3})$$

where  $\underline{Z}_i, \underline{x}_i$  denotes any (or a combination) of the dummy observations discussed in [Section 3](#).

In the conjugate case, the posterior distributions of  $\Psi_i$  and  $\Sigma_{\epsilon i}$  are of Normal and inverse-Wishart form, respectively. Formally, this implies that

$$\Psi_i | \Sigma_{\epsilon i}, \mathcal{D}_{iT} \sim \mathcal{N}(\bar{\Psi}_i, \Sigma_{\epsilon i} \otimes \bar{V}_i) \quad (\text{C.4})$$

$$\Sigma_{\epsilon i} | \Psi_i, \mathcal{D}_{iT} \sim \mathcal{IW}(\bar{S}_i, \bar{v}_i) \quad (\text{C.5})$$

where  $\mathcal{D}_{iT}$  denotes the information set spanned by observations for country  $i$  up to time  $T$ . The posterior mean of  $\Psi_i = \text{vec}(\Pi_i)$  is given by

$$\bar{\Pi}_i = (\bar{Z}_i' \bar{Z}_i)^{-1} \bar{Z}_i' \bar{x}_i, \quad (\text{C.6})$$

where  $\bar{Z}_i$  and  $\bar{x}_i$  denote the dummy-observation-augmented data matrices. Moreover,  $\bar{V}_i$  is simply

$$\bar{V}_i = (\bar{Z}_i' \bar{Z}_i)^{-1} \quad (\text{C.7})$$

The scale matrix of the posterior of  $\Sigma_{\epsilon i}$  is given by

$$\bar{S}_i = (\bar{x}_i - \bar{Z}_i \bar{\Pi}_i)' (\bar{x}_i - \bar{Z}_i \bar{\Pi}_i) \quad (\text{C.8})$$

and the posterior degrees of freedom are  $T + v_i$ . The conjugate nature of this prior implies that posterior distributions are available in closed-form.

### The Non-Conjugate / SSVS Case

Following [George \*et al.\* \(2008\)](#), we replace  $\Sigma_{\varepsilon i} \otimes \bar{V}_i$  in equation (C.4) by a  $v_i \times v_i$  matrix  $\bar{R}_i$ , where

$$\bar{R}_i = (\Sigma_{\varepsilon i}^{-1} \otimes (Z_i' Z_i) + \underline{R}_i^{-1})^{-1}. \quad (\text{C.9})$$

The mean of the conditional posterior is given by

$$\bar{\Psi}_i = \bar{R}_i (\underline{R}_i^{-1} \underline{\Psi}_i + \Sigma_{\varepsilon i}^{-1} \otimes (Z_i' x_i')) . \quad (\text{C.10})$$

The posterior degrees of freedom are still  $\bar{v}_i = T + \underline{v}_i$  and the posterior scale matrix is given by

$$\bar{S}_i = \underline{S}_i + (x_i - Z_i \bar{\Pi}_i)' (x_i - Z_i \bar{\Pi}_i). \quad (\text{C.11})$$

Finally, the conditional posterior of  $\delta_{ij}$  is distributed as Bernoulli,

$$\delta_{ij} | \delta_{i\bullet}, \Psi_i, \Sigma_{\varepsilon i}, \mathcal{D}_{iT} \sim \text{Bernoulli}(\bar{q}_{ij}) \quad (\text{C.12})$$

where the notation  $\delta_{i\bullet}$  indicates conditioning on all  $\delta_{ig}$  for  $g \neq j$  and the probability that  $\delta_{ij} = 1$  is given by

$$\bar{q}_{ij} = \frac{\frac{1}{\tau_{1j}} \exp(-\frac{\Psi_{ij}^2}{2\tau_{1j}})}{\frac{1}{\tau_{1j}} \exp(-\frac{\Psi_{ij}^2}{2\tau_{1j}}) \underline{q}_{ij} + \frac{1}{\tau_{0j}} \exp(-\frac{\Psi_{ij}^2}{2\tau_{0j}}) (1 - \underline{q}_{ij})}. \quad (\text{C.13})$$

## Appendix D Posterior Inference at the Global Level: The Implications and Advantages of Country-Specific Priors

The method described in Section 3 imposes priors exclusively at the individual country level. The main reason for local prior elicitation is computational. Furthermore, it is straightforward to show that placing the priors locally leads to the same priors on the global level scaled by the strength of the invoked trade links.

### Prior implications at the global level

The global implications of a prior imposed locally and the corresponding prior variances can be derived by substituting  $W_i x_t$  in equation (B.1),

$$A_i W_i x_t = B_i W_i x_{t-1} + \varepsilon_{it}. \quad (\text{D.1})$$

The prior variance-covariance matrices of  $A_i$  and  $B_i$  are given by

$$\underline{V}_{A_i} = \text{Var}[\text{vec}(A_i)] = \begin{pmatrix} 0_{k_i^2 \times k_i^2} & 0_{k_i^2 \times k_i k_i^*} \\ 0_{k_i k_i^* \times k_i^2} & \underline{V}_{\Lambda_{i0}} \end{pmatrix}, \quad (\text{D.2})$$

$$\underline{V}_{B_i} = \text{Var}[\text{vec}(B_i)] = \begin{pmatrix} \underline{V}_{\Phi_{i1}} & 0 \\ 0 & \underline{V}_{\Lambda_{i1}} \end{pmatrix}, \quad (\text{D.3})$$

where we assume without loss of generality that covariances between the blocks of coefficients equal zero. Consequently, the expressions for the variance-covariances matrices of  $\text{vec}(A_i W_i)$  and  $\text{vec}(B_i W_i)$  boil down to

$$\underline{V}_{A_i W_i} = \text{Var}[\text{vec}(A_i W_i)] = (W_i' \otimes I_{k_i}) \underline{V}_{A_i} (W_i' \otimes I_{k_i})', \quad (\text{D.4})$$

$$\underline{V}_{B_i W_i} = \text{Var}[\text{vec}(B_i W_i)] = (W_i' \otimes I_{k_i}) \underline{V}_{B_i} (W_i' \otimes I_{k_i})'. \quad (\text{D.5})$$

Here, for  $\Lambda_{i1}$ , it can easily be seen that the prior variance in country  $i$  corresponding to the coefficient associated with country  $j$ th endogenous variables is driven by the prior variance-covariance  $\underline{V}_{\Lambda_{i1}}$  and the trade weights in  $W_i$ .

Equations (D.4) and (D.5) imply that the variance-covariance matrices of  $G$  and  $H$  in equation (B.2) are given by

$$\underline{V}_G = \begin{pmatrix} \underline{V}_{A_1 W_1} & 0 & \cdots & 0 \\ 0 & \underline{V}_{A_2 W_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \underline{V}_{A_N W_N} \end{pmatrix}, \quad (\text{D.6})$$

$$\underline{V}_H = \begin{pmatrix} \underline{V}_{B_1 W_1} & 0 & \cdots & 0 \\ 0 & \underline{V}_{B_2 W_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \underline{V}_{B_N W_N} \end{pmatrix}, \quad (\text{D.7})$$

which are  $k^2 \times k^2$  matrices, respectively. Furthermore, equations (D.6) and (D.7) imply that the covariances between countries equal zero.

Finally, note that when defining conjugate priors locally the corresponding variances only need to be proportional to each other within a given country, since country models are estimated separately in the GVAR framework. This is in stark contrast to a global conjugate prior specification which would require variances of all equations in the system to be proportional to each other.

## Posterior simulation and computational issues

Constructing the prior at the local level (and thus leaving the fundamental GVAR structure untouched) facilitates the use of parallel computing. While the majority of the priors described directly come along with analytical posterior solutions, the SSVS prior for example does not. This implies that posterior simulation has to be carried out  $N + 1$  times, which might be computationally infeasible. Setting priors locally allows us to fully exploit parallel computing

and thus to carry out estimation even in the case posterior distributions are not available in closed form.

In an optimal parallel computing environment the computational speed can be increased  $c$  times, when spread across  $c$  central processing units (CPUs). However, in reality, only a fraction  $\varrho$  of some problem at hand can be parallelized. Amdahl's law (Rodgers, 1985) states that the maximum speedup gained by parallelization is given by  $1 / [\frac{\varrho}{c} + (1 - \varrho)]$ . In the GVAR case,  $\varrho$  is approximately one which implies that the GVAR framework is perfectly suited for exploiting gains from parallelization. This is in contrast to panel VARs (Korobilis, 2015) or large Bayesian VARs (Bańbura *et al.*, 2010) for which computational costs increase more strongly with the dimension of the estimation problem. Thus if the number of available CPUs equals the number of countries, the time needed to estimate a GVAR model using Bayesian methods approximately reduces to the time needed to estimate a single model. In practice, however, overhead costs typically arise. These costs are related to the transportation of the data from the host processor to the nodes. This is negligible relative to the overall estimation time, especially when we have to use simulation based methods. For our present application estimation of a model with  $p = p^* = 5$  lags takes between 30 minutes (conjugate specifications) to around two hours (non-conjugate/SSVS prior specifications) on a workstation with eight CPU cores and for 10,000 posterior draws.

Taking into account the computational advantages of such a modelling strategy, posterior inference is done locally, producing draws from the individual country posteriors for all countries in the sample. These draws are transformed using the usual GVAR algebra to produce valid draws from the (joint) global posterior of  $F$  and  $\Sigma_e$ , denoted by  $p(F, \Sigma_e | \mathcal{D}_T)$ , where  $\mathcal{D}_T$  denotes the available information set for all countries. Functions of the parameters like forecasts or impulse response functions can be easily calculated using Monte Carlo integration.

## Appendix E Forecast Measures

The  $f$ -step ahead predictive density of the GVAR model is given by

$$p(x_{t_0+f} | \mathcal{D}_{t_0}) = \int_{\tilde{F}} \int_{\Sigma_e} p(x_{t_0+f} | \tilde{F}, \Sigma_e, \mathcal{D}_{t_0}) p(\tilde{F}, \Sigma_e | \mathcal{D}_{t_0}) d\tilde{F} d\Sigma_e \quad (\text{E.1})$$

where  $\mathcal{D}_{T_0}$  denotes the data up to time  $T_0$  and  $\tilde{F} = (F_1, \dots, F_q)$ . Note that for conjugate priors, the one-step ahead predictive density is available in closed form. However, for  $f > 1$  we either have to resort to numerical methods or turn the problem at hand into a sequence of one-step ahead forecasts. The latter approach, known as the direct method, is employed for all conjugate prior distributions, whereas for non-conjugate distributions we use Monte Carlo integration to approximate the predictive density. This boils down to drawing from the country-specific posteriors and using the algebra outlined in Appendix B to obtain posterior draws of  $F$  and  $\Sigma_e$ . We construct the corresponding forecasts by iterating equation B.2 forward and sampling the corresponding errors from  $\mathcal{N}(0, \Sigma_e)$ . This procedure is repeated  $n_{sim}$  times.

As a point estimate, we use the mean of the predictive density described above. Evaluation of the point forecasts is based on the root mean square error (RMSE). The RMSE associated



with variable  $q$  is given by

$$\text{RMSE}_q = \sqrt{\frac{\sum_{t_0=T_0}^{T-f} (x(q)_{t_0+f}^O - \bar{x}(q)_{t_0+f})^2}{T - T_0 - f + 1}} \quad (\text{E.2})$$

where  $x(q)_{t_0}^O$  is the observed data corresponding to the elements in  $x_t$  and to variable  $q$ . The mean of the  $f$ -step ahead predictive density of variable  $q$  is denoted by  $\bar{x}(q)_{t_0+f}$ .

The log predictive score (LPS), as given in e.g. [Geweke & Amisano \(2010\)](#), is the predictive density given by equation (E.1) evaluated at the realized outcome.

$$\text{LPS}(x_{t_0+f}^O | \mathcal{D}_{t_0}) = \log p(x_{t_0+f} = x_{t_0+f}^O | \mathcal{D}_{t_0}) \quad (\text{E.3})$$

As noted above, for  $f > 1$  equation (E.3) has no closed form solution. Following [Adolfson et al. \(2007\)](#) we approximate the LPS using a multivariate normal density which is evaluated with posterior mean estimates from the predictive density. This second-order approximation is given by

$$\begin{aligned} \widehat{\text{LPS}}(x_{t_0+f}^O | \mathcal{D}_{t_0}) \approx & -0.5[k \log(2\pi) + \log |\bar{\Psi}_{t_0+f|t_0}| \\ & + (x_{t_0+f}^O - \hat{x}_{t_0+f|t_0})' \bar{\Psi}_{t_0+f|t_0}^{-1} (x_{t_0+f}^O - \bar{x}_{t_0+f|t_0})], \end{aligned} \quad (\text{E.4})$$

where  $\bar{\Psi}_{t_0+f|t_0}$  denotes the mean of the  $f$ -step ahead predictive variance-covariance matrix. We present variable-specific log predictive scores calculated by integrating out the effect of other variables in the system. Under the assumption of multivariate normality, the LPS associated to variable  $q$  can be calculated by deleting the rows (and columns) corresponding to variable  $g \neq q$  from  $x_{t_0+f}^O$ ,  $\bar{x}_{t_0+f|t_0}$  and  $\bar{\Psi}_{t_0+f|t_0}$ , respectively.